NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2967

AN ANALYSIS OF THE POWER-OFF LANDING MANEUVER IN TERMS OF THE CAPABILITIES OF THE PILOT AND THE AERODYNAMIC CHARACTERISTICS OF THE AIRPLANE

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SUMMARY

An analysis of the power-off landing maneuver is presented in which an attempt is made to consider the human capabilities of the pilot in addition to the aerodynamic characteristics of the airplane. Assumptions are made that the pilot's judgment of distance may be inaccurate by a certain factor E_h and that a time delay t_r occurs between a decision to correct the airplane attitude and the time that such correction is effected. These parameters E_h and t_r are included in the landing calculations to modify the optimum landing paths derived from purely aerodynamic considerations so as to give them inherent safety margins. The corresponding determination of a minimum safe initial glide speed and the definition of a region within which the pilot should fly in order to make a safe landing in a minimum distance are described. Several calculated results based on assumed values of E_h and t_r are presented. The results obtained from the present analysis show the desirability of future research to determine accurate values of E_h and t_r.

INTRODUCTION

The manner in which an airplane is landed in a region of limited extent depends not only on the physical and aerodynamic characteristics of the airplane but also on the knowledge and skill of the pilot. In previous analyses of the landing maneuver (see, for example, reference 1) the effects of variation in the characteristics of the airplane have been carefully assessed but the question of whether a pilot could be expected to fly an airplane along any of the calculated flight paths has not been specifically considered.

Some margin for pilot deviation from a predetermined path is given in the calculations of reference 1 by assuming that at no point in the landing flare does the lift coefficient exceed 0.85 of the maximum lift coefficient. This condition was derived from the flight records given

in reference 2. The general applicability of this specific "safety factor" to other airplanes is open to question because it is not related in any very direct way to pilot capabilities. Similar safety factors are found in other analyses.

The arbitrary application of such a safety margin in order to allow for the effects of pilot judgment probably gives practical results for airplanes having small wing loadings or for airplanes having design parameters similar to the airplanes from which the safety factors were derived. Since, with the increasing wing loading of modern airplanes, pilot judgment becomes of increasing importance, a more careful estimate of the effects of pilot judgment on the landing maneuver than has previously been made seems necessary. Such an estimate is particularly desirable to determine whether an airplane of unconventional design and high wing loading can be landed while maintaining predetermined allowances for pilot judgment and, if so, the size of the landing field required.

In the present paper, an analysis of the power-off landing maneuver is presented in which an attempt is made to consider both the aerodynamic characteristics of the airplane and the human capabilities of the pilot. A purely aerodynamic analysis results in one aerodynamically optimum landing path. Because of variations in judgment, however, a pilot cannot follow this or any other given landing path every time. The effects of pilot judgment must be included in any realistic analysis of the landing maneuver. In this paper, the type of pilot judgment considered is that of a pilot with previous flight experience, although not necessarily flight experience in any particular airplane under discussion. Pilot judgment is believed to influence primarily the orientation of the glide path preceding the landing flare and the height at which the flare is begun. It is assumed that the pilot's judgment of distance may be inaccurate by a certain factor E_h and that a time delay t_r occurs between a decision to correct the airplane attitude and the time that such correction is effected. These factors E_h and t_r are included in the landing calculations to modify the optimum landing paths derived from purely aerodynamic considerations so as to give them inherent safety margins. The calculations resulted in the determination of a minimum safe initial gliding speed and in the definition of a region within which the pilot should fly in order to land the airplane safely. The extent of this region is coupled with ground-run calculations and used to determine the corresponding minimum size of the landing field. Future research is needed for the accurate determination of E_h and t_r.

SYMBOLS

C_D drag coefficient of airplane, based on wing area

C_{D_o}' drag coefficient of airplane at base of ideal flare

C_{D_b}	drag coefficient at start of braking run
C_L	lift coefficient of airplane, based on wing area
C_{L_o}	lift coefficient of airplane at base of flare
C_{L_o}'	lift coefficient of airplane at base of ideal flare
C_{L_b}	lift coefficient of airplane at start of braking run
C_{L_g}	lift coefficient of airplane during straight glide
a_t	acceleration of airplane tangential to flare path, positive in direction of flight, feet per second per second
$\bar{c}$	mean aerodynamic chord of airplane wing, feet
D_b	drag force at start of braking run, pounds
E_h	relative error in height estimation
E_s	relative error in estimating horizontal distance traveled during straight glide
E_v	relative error in wind velocity
E_w	relative error in estimating sinking speed w
g	acceleration due to gravity, 32.17 feet per second per second
h	flare-path height above runway, feet
h'	ideal flare-path height above runway, feet
h^*	nondimensional height parameter $\left(\frac{gh'}{{V_o'}^2} \text{ or } \frac{gh}{{V_o'}^2}\right)$
h_1'	height of flare path at beginning of ideal flare, feet
h_{g_s}	height of airplane at start of straight glide, feet
L_b	lift force at start of braking run, pounds

L/D	ratio of lift force to drag force
$(L/D)_o$	lift-drag ratio at base of flare
$(L/D)_o'$	lift-drag ratio at base of ideal flare
$(L/D)_g$	lift-drag ratio during straight glide
R	radius of curvature of flare path, feet
S	wing area, square feet
s	flare-path length measured from base of flare, positive in direction of flight, feet
s'	horizontal distance along ideal flare path, feet
s^*	nondimensional flare-path parameter $\left(\frac{gs}{V_o'^2} \text{ or } \frac{gs'}{V_o'^2}\right)$
s_b	length of no-wind braking run after landing, feet
s_{basic}	horizontal distance along landing path from 50-foot-altitude station to end of ground run, feet
s_g	horizontal distance traveled during straight glide in line with runway, feet
s_T	length of minimum-distance safe-landing field, feet
t_g	time required for straight glide in line with runway, seconds
t_l	time required to perform entire landing maneuver, seconds
t_r	time delay in taking corrective action during flare, seconds
V	airspeed, feet per second
V_o	airspeed at base of flare (stalling speed), feet per second
V_o'	airspeed at base of ideal flare (stalling speed), feet per second

V_1	airspeed at start of flare path, feet per second
V^*	velocity ratio $\left(V/V_o'\right)$
V_b	airspeed at start of braking run, feet per second
V_g	gliding speed, feet per second
v	wind velocity, feet per second
w	vertical velocity, feet per second
W	gross weight of airplane in landing condition, pounds
W/S	wing loading, pounds per square foot
γ	flare-path angle measured from horizontal plane, negative downward, radians
γ_1'	flare-path angle at start of ideal flare, radians
γ_g	glide-path angle, radians
Δh_1	height correction to allow for time delay, feet
Δh_2	error in height estimation, feet
Δh	total height correction, feet $(\Delta h_1 + \Delta h_2)$
Δh_{g_s}	error in height at beginning of straight glide, feet
Δs	total horizontal error in spacing of limit glide paths on either side of minimum-distance safe-landing glide path, feet
μ	ground friction coefficient of airplane
ρ	mass density of atmosphere, slugs per cubic foot

Subscript:

max	maximum

ANALYSIS AND DISCUSSION

The landing maneuver, as herein analyzed, consists of a steady glide followed by a landing flare and a ground run. In the present analysis, the calculations for the optimum aerodynamic landing flare are given first in order to determine the basic nondimensional parameters. Following this development, a section is given in which the effect of pilot judgment on the landing maneuver is discussed. The many considerations involved in performing the maneuver are discussed and two factors are chosen which are considered to be predominant, (1) the judgment of height and distance and (2) a time lag associated with pilot and airplane response. Numerical constants, believed to be indicative of these two factors, are inserted into the aerodynamic landing calculations to determine the minimum safe glide speed and the minimum necessary extent of a flare region within which the pilot should fly in order to land the airplane safely. Next, standard ground-run calculations are given. In the final section the effect of pilot judgment on properly orienting the glide path in space with relation to the landing field is taken into account. The ground run and glide-path orientation are then combined with the limits of the landing flare region to give a method of analysis for obtaining an estimate of the minimum length of the landing field required to land an airplane safely.

Landing Flare

A landing flare may be defined as a maneuver which changes the flight condition of an airplane from that of a steady glide at a speed greater than the stalling speed to horizontal flight at zero altitude with zero vertical acceleration. The excess above stalling speed is maintained in the glide so that the lift developed during the flare will be in excess of the weight and a resultant force is created which will reduce the sinking speed at ground contact to zero. The speed of emergence from the flare should be at least as great as the stalling speed. If the speed of emergence from the flare is in excess of the stalling speed, the airplane can float parallel to the ground at a lift coefficient below the maximum until stalling speed is reached.

Ideal landing flare.- The aerodynamically ideal flare path (which in this paper is referred to as the ideal flare path) is here considered that flare path which will accomplish the landing-flare maneuver in the shortest possible horizontal distance. For a given value of the difference between glide speed and stalling speed, the distance required to decelerate a particular airplane to the stalling speed will be least when the decelerating force is greatest. This condition can be shown to occur when the flare is executed at maximum lift coefficient from

a glide at a certain minimum glide speed (reference 3) such that the speed of emergence from the flare corresponds to the stalling speed. The ideal landing maneuver, then, consists of a steady glide at the optimum approach speed followed by an ideal flare.

The problem of determining the equations for the previously defined ideal flare path has been treated in reference 3. The following treatment, based on unpublished work by J. W. Wetmore, derives similar relations in a somewhat more convenient form.

Consider an airplane flying along a curved path such as shown in figure 1. If the airplane is flying at maximum lift, the lift and drag coefficients are constant, although the lift and drag forces are changing because of the decrease in speed. If the slope of the flare path is assumed sufficiently small so that the cosine of the flare-path angle is equal to unity and the sine of the angle γ is equal to γ, a summation of the forces normal to the flare path gives

$$\frac{W}{g}\frac{V^2}{R} + W = C_{L_O}{}'\frac{\rho}{2}SV^2$$

so that

$$\frac{1}{R} = \frac{gC_{L_O}{}'\rho S}{2W} - \frac{g}{V^2}$$

But the curvature $1/R$ is equal to the rate of change $d\gamma/ds'$ of the flare-path angle with respect to the distance along the flare path. Therefore,

$$\frac{d\gamma}{ds'} = g\left(\frac{C_{L_O}{}'\rho S}{2W} - \frac{1}{V^2}\right)$$

Multiplying by ${V_O{}'}^2/g$ gives

$$\frac{{V_O{}'}^2}{g}\frac{d\gamma}{ds'} = \frac{C_{L_O}{}'\rho S{V_O{}'}^2}{2W} - \left(\frac{V_O{}'}{V}\right)^2$$

Since the lift coefficient is constant,

$$C_{L_o}{}' \frac{\rho}{2} S V_o{}'^2 = W$$

therefore,

$$\frac{V_o{}'^2}{g} \frac{d\gamma}{ds'} = 1 - \left(\frac{V_o{}'}{V}\right)^2 \tag{1}$$

Summing the tangential forces gives, after dividing throughout by the airplane mass,

$$a_t = -\frac{g}{W}\left(C_{D_o}{}' \frac{\rho}{2} S V^2 + W\gamma\right)$$

but

$$a_t = V \frac{dV}{ds'} = \frac{1}{2} \frac{d(V^2)}{ds'}$$

therefore

$$\frac{d(V^2)}{ds'} = -\frac{2g}{W} C_{D_o}{}' \frac{\rho}{2} S V^2 - 2g\gamma$$

In the first term on the right, replacing $C_{D_o}{}'$ by $C_{L_o}{}'/(L/D)_o{}'$ and multiplying by $V_o{}'^2/V_o{}'^2$ gives

$$\frac{d(V)^2}{ds'} = -\frac{2gC_{L_o}{}' \frac{\rho}{2} S V^2 V_o{}'^2}{W(L/D)_o{}' V_o{}'^2} - 2g\gamma$$

Since the lift coefficient throughout the ideal flare is constant, replacing $C_{L_o}{}' \frac{\rho}{2} S V_o{}'^2$ by W gives

$$\frac{d(V^2)}{ds'} = -\frac{2g}{(L/D)_o{}'}\left(\frac{V}{V_o{}'}\right)^2 - 2g\gamma \tag{2}$$

The distance s' and velocity V may be rendered nondimensional by the following substitutions:

$$\frac{gs'}{V_o'^2} = s^* \qquad ds' = \frac{V_o'^2}{g} ds^*$$

$$\frac{V}{V_o'} = V^* \qquad d(V^2) = V_o'^2 d\left(V^{*2}\right)$$

Thus, the nondimensional forms of equations (1) and (2), respectively, become

$$\frac{d\gamma}{ds^*} = 1 - \frac{1}{V^{*2}} \tag{3}$$

and

$$\frac{d\left(V^{*2}\right)}{ds^*} = -2\left[\frac{V^{*2}}{(L/D)_o'} + \gamma\right] \tag{4}$$

Also

$$\frac{d\gamma}{d\left(V^{*2}\right)} = \frac{d\gamma/ds^*}{d\left(V^{*2}\right)/ds^*}$$

or

$$\frac{d\gamma}{d\left(V^{*2}\right)} = -\frac{1 - \frac{1}{V^{*2}}}{2\left[\frac{V^{*2}}{(L/D)_o'} + \gamma\right]} \tag{5}$$

Writing $dh' = \gamma\, ds'$ and letting $h^* = \frac{gh'}{V_o'^2}$ so that $dh^* = \frac{g}{V_o'^2} dh'$ gives

$$dh^* = \gamma\, ds^*$$

and

$$\frac{dh^*}{d\left(V^{*2}\right)} = -\frac{\gamma}{2\left[\frac{V^{*2}}{(L/D)_o{}'} + \gamma\right]} \tag{6}$$

Also

$$\frac{ds^*}{d\left(V^{*2}\right)} = -\frac{1}{2\left[\frac{V^{*2}}{(L/D)_o{}'} + \gamma\right]} \tag{7}$$

Equations (5), (6), and (7) are the nondimensional differential equations which determine the ideal flare path. If equation (5) can be evaluated in terms of V^{*2}, then, by substituting the values of γ thus obtained into equations (6) and (7), these equations can be solved by simple integration. Equation (5) is a nonlinear differential equation. One method of solution that suggests itself is use of a step-by-step integration process beginning at the point $V^{*2} = 1$, $\gamma = 0$ in order to satisfy the boundary conditions. Unfortunately, near this point determination with sufficient accuracy of the variation of γ with V^{*2} becomes difficult. Equation (5), however, can be linearized by expanding the right-hand side into a power series in γ and neglecting all terms containing powers of γ higher than the first. The linearized form of equation (5), $\frac{d\gamma}{dV^{*2}} + r_1\gamma = r$, where $r = \frac{-\frac{1}{2} + \frac{1}{2}V^{*2}}{V^{*2}/(L/D)_o{}'}$ and $r_1 = \frac{r}{V^{*2}/(L/D)_o{}'}$, is a differential equation which can be integrated readily by standard methods. The result is as follows:

$$\gamma = e^{\int_1^{V^{*2}} -r_1 dV^{*2}} \int_1^{V^{*2}} r e^{\int_1^{V^{*2}} r_1 dV^{*2}} dV^{*2} \tag{8}$$

This integral is evaluated by means of Simpson's rule. Then, the values of γ in terms of V^{*2} can be substituted into equations (6) and (7) to obtain solutions by numerical integration. The results of the solutions of equations (5), (6), and (7) are given in graphical form

in figures 2 and 3. Figure 2 is a plot of V/V_o' and γ against s^* for a wide range of values of $(L/D)_o'$. Figure 3 is a plot of h^* against s^* for the same values of $(L/D)_o'$. By the use of these graphs, the shape of the ideal flare path for any airplane within the range of $(L/D)_o'$ values presented may be calculated. The fact that equations (5), (6), and (7) do not contain the airplane weight, wing area, or maximum lift coefficient explicitly means that the ideal landing flare is independent of these quantities except insofar as they determine the value of V_o' and thereby control the linear scale of the ideal landing flare. The only parameter that affects the nondimensional shape of the ideal flare path is the airplane lift-drag ratio at maximum lift.

In order to determine completely the ideal flare path, not only the shape of the flare path but also the point of entrance into the ideal flare must be found. The point of entrance into the ideal flare is found by applying conditions of continuity to the motion of the airplane. In order to avoid discontinuities in the motion of the airplane center of gravity, it is assumed (1) that the flight-path angle in the glide is the same as that at the point of entrance into the flare and (2) that the speed along the glide path is the same as the speed at the point of entrance into the flare. These conditions mean that the path of steady glide of the airplane is tangent to the ideal flare path at the point where the speed along the flare path is the same as the speed along the glide path. In general, the glide-path angle for a steady glide is a single-valued function of the speed along the glide path determined from the lift-drag polar curve for the airplane. That is,

$$\gamma_g = \frac{1}{(L/D)_g} \qquad C_L = \frac{C_{L_o}'}{(V/V_o')^2}$$

A typical curve for the relation between γ_g and V/V_o' is given by the curve labeled γ_g in figure 4. A different relationship between γ and V/V_o' exists for the ideal flare because the velocity along the flare path is not constant. An example of the variation of γ and V/V_o' for a case where $(L/D)_o'$ is 4.08 is given by the curve so labeled in figure 4. These two curves of γ and γ_g plotted against V/V_o' are seen to intersect at one point. Through the relationships indicated in figures 2 and 3, the values of γ and V/V_o' corresponding to this point of intersection determine the point of entrance into the ideal flare.

These curves of γ and γ_g against V/V_o' have another special significance. They provide an inherent limitation to the values of the parameters which may be used in this analysis. If the two curves intersect, it means that for the parameters used it is aerodynamically possible to enter the calculated flare path from a state of steady glide. The data presented in figures 2 and 3 show that the value of V/V_o' corresponding to a given value of γ increases as $(L/D)_o'$ decreases. If $(L/D)_o'$ for a given airplane were sufficiently low, the curve of γ against V/V_o', for example, as presented in figure 4 would be moved sufficiently to the right so as never to intersect the curve of γ_g against V/V_o' corresponding to steady-glide conditions. If the two curves do not intersect, it is impossible to get enough speed in the steady glide to make the calculated landing flare and the present analysis would fail to indicate the possibility of a safe landing with power off. This method of joining the glide path to the ideal flare path involves an instantaneous change in lift coefficient and, hence, in angle of attack at the point of entrance into the flare. An allowance for this condition is discussed in the following section of the analysis.

Because the ideal nondimensional glide path and flare path are completely determined by the airplane polar curve of C_L against C_D, such variables as wing loading or glide-path sinking speed are irrelevant to the problem of calculating the ideal landing maneuver except insofar as they are indicative of the stalling speed and lift-drag ratio at maximum lift.

In the execution of an ideal flare at $C_{L_{max}}$, the point at which the normal component of acceleration is greatest is that point at which the value of V^* is greatest. In fact, the normal component of acceleration equals $V^{*2}g$. The point of maximum normal acceleration will therefore generally occur at the point of entrance into the flare, that is, the point in the flare where V^* is a maximum value. For any given value of γ, the value of V^* decreases as $(L/D)_o'$ increases. Consequently, the airplanes having the lowest values of $(L/D)_o'$ would be expected to have the highest values of V^* in the flare and thus to experience the highest normal accelerations. In an example to be discussed subsequently, an airplane having a value of $(L/D)_o' = 1.25$ at maximum lift entered the ideal flare with a value of V^* of 1.487. The corresponding value of normal acceleration for the airplane is 2.21g. The value of this acceleration in the landing flare is larger than the values to which pilots are accustomed, but the values given are for an extreme condition $\left((L/D)_o' = 1.25\right)$. The normal accelerations corresponding to higher values of $(L/D)_o'$ would be considerably less and

will be substantially reduced by the allowances to be made for pilot judgment. Therefore, in general, the normal accelerations involved in the use of this analysis would not seem to constitute a limiting factor in the landing maneuver. It is interesting to note that the value of the normal acceleration depends upon the value of V^* and thus is independent of the wing loading. The lower limit of the lift-drag ratio is probably determined by the condition that the speed required upon entry into the flare shall be no greater than that of the maximum safe gliding speed as determined by structural considerations.

Effects of pilot judgment on the landing flare.- The analysis thus far has presented an aerodynamically ideal flare path. For a pilot to attempt to use an ideal flare in a landing, however, would be extremely dangerous, because once an ideal flare is begun, the pilot could not deviate at all from it and still land safely. If the pilot started such a flare and found himself too low, he would be unable to pull up since he would already be flying at $C_{L_{max}}$. Consequently, he would fly into the ground. If, on the other hand, the pilot found himself too high, he would not have enough speed margin to finish the longer flare path and he would stall out at some distance above the ground. Nevertheless, since minimum landing distances are of primary interest, the ideal flare is useful, from the point of view of determining the important aerodynamic variables, as a basis for further discussion and analysis. Since, as was stated previously, a pilot practically never duplicates any flight path exactly in making a landing, the extent of a region such that a safe landing may be completed from any point within the region should be defined. The minimum necessary extent of such a region will be determined largely by the variability of the judgment and responses of the pilot in attempting to fly an idealized (not necessarily the "ideal") flare path.

The next step in the analysis is the determination of the most important elements of judgment involved in making a landing. Aside from the yawing or rolling attitude of the aircraft, the quantities which are needed to specify the flight conditions of the airplane in a flare path are the airspeed, the glide-path angle, the normal component of acceleration, and the position with respect to the ground. If the glide speed at the beginning of the flare is presumed to be adequate, the actual instantaneous value of the airspeed at any point in the flare is normally not noted by the pilot. Apart from the horizontal position, the suitability during the landing flare of the flight condition of the airplane at any point along the flare is judged by the pilot through simultaneous estimates of sinking speed, normal acceleration, and height.

In order to obtain a possible simplification of the problem, the relative importance and necessary accuracy of these three types of judgment should be determined. The flare path can be calculated if any one of the quantities h, dh/dt, or d^2h/dt^2 is known as a function of time. The airplane presumably can be landed safely if its altitude at a particular point is within an increment Δh of the desired altitude at that point. The corresponding necessary accuracy of height judgment is simply $\frac{\Delta h}{h} = E_h$. The necessary relative accuracy in judgment of the sinking speed E_w can be estimated from the following considerations: The deviation in sinking speed from the desired value is assumed to be oscillatory in nature with a characteristic frequency n corresponding to the time interval between successive judgments by the pilot. The order of magnitude of the time interval $1/n$ may be expected to be the same as the response time of the pilot-airplane combination. If $E_w w$ is the maximum allowable deviation in sinking speed from the desired value, then this oscillatory deviation can be expressed in the form $E_w w \sin 2\pi nt$. The corresponding maximum departure in height from the desired value is obtained by integrating this incremental velocity over a half cycle. The result when the variation of w over a period of time $1/2n$ is negligible is $E_w w/\pi n$. If this expression is equated to the maximum allowable error in height Δh the result is $\Delta h = E_h h = \frac{E_w w}{\pi n}$. The relative magnitude of E_h and E_w corresponding to equal deviations in height is then $\frac{E_h}{E_w} = \frac{w}{\pi nh}$.

The value of $w/\pi nh$ calculated for typical flare paths to be presented subsequently was found to be of the order of magnitude of 0.1. Even though the oscillations in sinking speed may be somewhat irregular and the characteristic frequency n somewhat larger than the response time of the airplane, nevertheless, in most cases accurate judgment of the sinking speed is likely to be much less important than accurate judgment of height.

By a similar procedure the accuracy of judgment of vertical acceleration is seen to be even less important than accurate judgment of sinking speed. The judgment of height, then, seems to be the most critical element of pilot judgment.

In addition to an allowance for errors in judgment, another type of allowance must be made in order to arrive at an estimate of the extent of a safe-landing flare region. This allowance is associated with a time delay. Once a pilot has formed a judgment as to the unsuitability of a particular flight condition, some period of time must naturally elapse before the appropriate corrective action can become effective.

The time consumed in obtaining corrective action is not entirely a function of pilot reaction time but may depend largely upon the response characteristics of the airplane. This time-delay factor may be considered, in effect, an additional height error, since the altitude loss due to a time delay may be expressed as

$$\Delta h_1 = t_r w = -t_r \gamma V$$

This relationship indicates that, at any point along the flare path, the pilot may fly tangentially to the flare path for the full time t_r before changing the attitude of the airplane; however, the point at which the time delay is largest is at the entrance into the flare where a large change in angle of attack is required to change from a lift coefficient during glide to a lift coefficient near maximum lift for the flare path. At points farther along the flare path, the time delay is less since the change in angle of attack associated with small adjustments would not be so large once the flare path has been established and, in any case, a pilot would not be likely to reduce the normal component of acceleration completely to zero, as would be the case if he departed from his intended path along a tangent line. In accordance with these considerations, Δh_1 has been assumed to be proportional to the height above the ground; that is,

$$\Delta h_1 = \frac{h'}{h_1'}\left(-t_r \gamma_1 V_1\right) \tag{9}$$

which gives a maximum allowance for the time delay at the beginning of the flare.

Physiological measurements of minimum perceptible differences in various stimuli such as pitch of sound, intensity of light, and many others indicate that the minimum perceptible difference in the stimulus is proportional to the stimulus. That is, if $\Delta\eta$ represents an increment on the sensitivity scale, $\Delta\eta = K\frac{\Delta I}{I}$, where I is the absolute physical intensity or magnitude of the stimulus; that is, $\eta = K \log I$. (See reference 4.) Human sensitivity to height can reasonably be assumed to follow a similar relation. The actual error Δh_2 in estimating height above the ground is therefore assumed to be proportional to the height estimated. The height to be estimated along a desired safe-landing flare path includes the ideal height h', the response-time altitude loss Δh_1, and the altitude-estimation error Δh_2 itself. These allowances are included in order that, with maximum height-estimation deviation

and time delay, the pilot will at worst find himself on the ideal flare path and should be able to land safely. Corresponding points along different flare paths are taken as points having equal slope. Accordingly, the altitude-estimation-factor allowance may be expressed as

$$\Delta h_2 = E_h\left(h' + \Delta h_1 + \Delta h_2\right)$$

For purposes of computation, this equation may be expressed as

$$\Delta h_2 = \frac{E_h}{1 - E_h}\left(h' + \Delta h_1\right) \tag{10}$$

Minimum-distance safe-landing flare region.- In this section the determination is made of the extent of a landing flare region between two limit flare paths such that a safe landing may be made from any point within this region. In order to set up the limit flare paths, it is necessary to define the nature of the maximum deviation that a pilot is likely to make in trying to fly a given flare path. This given flare path (called herein the minimum-distance safe-landing flare path) is derived from the assumption that the flare path corresponding to a maximum deviation toward the ground from this intended flare path shall be an ideal flare path. Since the pilot is just as likely to fly above an intended path as below it, the upper limit of the region within which the pilot may fly the airplane is found by assuming a corresponding margin of error above the minimum-distance safe-landing flare path. The upper-limit and lower-limit flare paths enclose a region to be known as the minimum-distance safe-landing region.

At this point, a discussion of the practical significance of the minimum-distance safe-landing flare region should be of interest. Although detailed calculations for a particular flare path are given (the minimum-distance safe-landing flare path), the pilot cannot be expected to follow this or any other flare path precisely. If the characteristics of the airplane and appropriate values of E_h and t_r are assumed to be known, however, the calculations do indicate to the pilot the altitude at which he should try to begin the flare and the minimum safe value of the glide speed immediately preceding the flare. If he uses this information at the beginning of the flare, a pilot should be able to complete the execution of the flare maneuver in a normal manner because appropriate margins for variations in judgment are inherent in the setting up of the initial conditions. Any such flare should fall within the limits of the minimum-distance safe-landing flare region.

In order to determine the characteristics of the minimum-distance safe-landing flare path, the correction for height error must be applied to the ideal flare path. Assume a flare in which the speed of entrance into the flare and the initial glide-path angle are the same as in the ideal flare path, but the altitude of the flare is greater at every point than the corresponding point for the ideal flare path by the allowable margin of height error. Consider the corrected flare path as the curve defined by the relation that points of equal slope occur at height h' along the ideal flare path and at $h' + \Delta h$ along the corrected flare path. If the ideal flare path is given in the form

$$\gamma = f(h') \tag{11}$$

the curve for h, defined by the relation

$$\gamma = f(h - \Delta h) \tag{12}$$

may be considered the flare path corrected for height-estimation error. The variable Δh in equation (12) is the height allowance for pilot error and may be analyzed as

$$\Delta h = \Delta h_1 + \Delta h_2$$

where Δh_1 and Δh_2 are given in equations (9) and (10). Thus, the expression for the height at any particular flare-path slope of the path the pilot should attempt to fly takes the form

$$h = h' + \Delta h_1 + \Delta h_2 \tag{13}$$

Substituting the values of Δh_1 and Δh_2 (equations (9) and (10)) into equation (13) produces the following relation:

$$h = h' \frac{h_1' - t_r \gamma_1 V_1}{h_1'} \frac{1}{1 - E_h} \tag{14}$$

Equation (14) is in the form $h = h'$ times a constant; that is,

$$h = Kh'$$

where

$$K = \frac{h_1' - t_r\gamma_1 V_1}{h_1'} \frac{1}{1 - E_h} \tag{15}$$

In order to complete the expression for the flare path corrected for height error, an expression must be found for the horizontal distance s. For the ideal flare path, $ds' = \frac{dh'}{\gamma}$. Therefore, for the corrected flare path, $s = \int \frac{dh}{\gamma}$. Since $h = Kh'$, $s = K\int \frac{dh'}{\gamma} = Ks'$. Substituting for K (equation (15)) yields

$$s = s' \frac{h_1' - t_r\gamma_1 V_1}{h_1'} \frac{1}{1 - E_h} \tag{16}$$

Equations (14) and (16) give the shape of the minimum-distance safe-landing flare path which the pilot should try to fly. If the ideal flare path is the lower limit of the minimum-distance safe-landing region and the minimum-distance safe-landing flare path is in the center of the region, then the upper-limit flare path would be the flare path which a pilot would fly if he overestimated his altitude through the flare by an amount equal to the maximum allowable height error, and this flare path would be above the minimum-distance safe-landing flare path by an amount equal to $\frac{h}{1 - E_h}$. This longer flare path will be denoted as the maximum flare path. Thus,

$$h_{max} = h \frac{1}{1 - E_h}$$

or

$$h_{max} = Kh' \frac{1}{1 - E_h}$$

Substituting for K gives

$$h_{max} = h' \frac{h_1' - t_1\gamma_1 V_1}{h_1'}\left(\frac{1}{1 - E_h}\right)^2 \tag{17}$$

Now, by analogy to the development used for equation (16),

$$s_{max} = \int \frac{dh_{max}}{\gamma} = \frac{1}{1 - E_h} K \int \frac{dh'}{\gamma} = \frac{1}{1 - E_h} Ks'$$

Substituting for K (equation (15)) gives

$$s_{max} = s' \frac{h_1' - t_r\gamma_1 V_1}{h_1'}\left(\frac{1}{1 - E_h}\right)^2 \tag{18}$$

The speed along the maximum or upper-limit flare path must be such that the values of V/V_o' along the longer flare path are at least equal to or greater than the values of V/V_o' at points having equal values of γ along the ideal flare path. If this condition is not met, the airplane will not have enough speed to complete the maximum flare path and will stall before reaching the ground. Because the values of V/V_o' cannot be specified independently of h for a given value of γ, the variation of V/V_o' along the maximum flare path must be determined. The relations necessary for computing the speed variation along an arbitrary flare path are derived and presented in the appendix. These equations are

$$\frac{dV}{ds} = -\frac{g}{\sqrt{W}}\left[\frac{C_D \frac{\rho}{2} S}{\sqrt{C_L \frac{\rho}{2} S - \frac{W(d\gamma/ds)}{g}}} + \gamma\sqrt{C_L \frac{\rho}{2} S - \frac{W(d\gamma/ds)}{g}}\right] \tag{19}$$

$$V = \int \frac{dV}{ds} ds$$

where

$$C_L = \frac{W/S}{\rho/2}\left(\frac{1}{V^2} + \frac{1}{gR}\right)$$

$$R = \frac{1}{d\gamma/ds}$$

The necessary speed at the entrance to the maximum flare is found by beginning at the base of the flare path with the stalling speed and using a step-by-step process to solve equation (19) for increments of ds. This procedure gives a curve of V^* plotted against γ for the maximum flare path. The value of V^* at the point of intersection of this curve with the curve of V^* against γ derived from the airplane polar determines the value of the glide speed at the start of the maximum flare. This speed is the glide speed which the pilot should use. The increases in steady glide speed necessary to permit overshooting of the intended path will decrease rather than increase the minimum turning radius in a vertical plane. This increase in the steady glide speed produces a conservative result since it will actually permit a small undershooting of the lower-limit flare path derived for the lower glide speed.

The results of typical flare-path calculations showing the effects of variations of E_h, t_r, wing loading, and lift-drag ratio are shown in figures 5 to 9. In order to carry out these calculations, various typical values of E_h and t_r had to be assumed. The physical significance of t_r is fairly obvious. It is the time interval between the instant when the pilot desires to change the attitude of the airplane and the instant when such a change has been completed. For the types of airplanes considered in the following calculations, t_r was assumed to be of the order of magnitude of 0.5 second to 1 second. As stated previously, E_h is the relative accuracy with which the pilot can estimate height. A value of $E_h = 0$ would correspond to absolute accuracy of judgment and a value of $E_h = 1$ would correspond to a pilot with no judgment at all. A reasonable value of E_h would seem to be approximately 1/4. For example, if a pilot believes he is 100 feet from the ground and E_h is taken as 1/4, the airplane may be either $1 - E_h$ times the estimated height or $\frac{1}{1 - E_h}$ times the estimated height, that is, between 75 feet and 133 feet. Figure 5 shows, as expected, that the flare-path size increases with increasing values of E_h and t_r. As the flare-path dimensions increased in comparison

with the ideal flare path, it was found necessary to increase progressively the speed of entrance into the flare. Figures 6 and 7 show the results of variation of wing loading with fixed values of E_h of 1/4 and t_r of 0.5 second. Figure 6 shows that the flare-path dimensions increase with an increase in wing loading. Figure 7, which presents the same flare paths as figure 6 in nondimensional form, shows that on a nondimensional basis the flare-path dimensions actually decrease with an increase in wing loading. This effect is associated with the decrease in relative importance of the effect of t_r, as compared with E_h, with increasing stalling speed. The height allowance associated with E_h is a constant proportion of the corresponding dimension of the ideal flare path. Consequently, this allowance varies as the square of the stalling speed. The height allowance associated with t_r, however, varies only as the first power of the stalling speed. Therefore, as the stalling speed is increased, the height allowance due to height-estimation error E_h increases more rapidly than that due to response time lag t_r.

The effect of the variation of the lift-drag ratio is shown in figure 8. The effect of decreasing lift-drag ratio in shortening the extent of the flare is still present after allowances are made for $E_h = \frac{1}{4}$ and $t_r = 0.5$ second but is less marked than in the case of the corresponding ideal flare paths shown in figure 3. In figure 9, landing flare paths for two delta-wing configurations (for which wind-tunnel data were available) with a wing loading adjusted to give the same stalling speed (based on $C_{L_{max}}$) indicate again that the configuration having the lower lift-drag ratio has the shorter flare. The sinking speeds in the steady glide preceding the flare for delta wings 1 and 2 were 36.4 feet per second and 73.8 feet per second, respectively. A sinking speed as high as that for delta wing 2 might ordinarily be considered unsafe. Along the ideal flare for delta wing 2 the sinking speed at 50 feet is 56 feet per second but, along the minimum-distance safe-landing flare path, the sinking speed at 50 feet of altitude is reduced to 37 feet per second and, of course, continues to decrease to zero as the ground is approached. The inclusion of the factors E_h and t_r in the calculated flare path is seen to reduce the sinking speed to a reasonable value at an altitude sufficiently great to allow the pilot time for making final adjustments before contacting the ground.

At this point, a discussion of the effect of E_h and t_r on the intersection of the curves of γ and γ_g against V^* which determine the point of entrance into the flare is of interest. (See fig. 4.) Because of the increased dimensions of the flare path associated with the margins E_h and t_r, it is, in general, found necessary to increase

somewhat the values of V^* associated with a particular value of γ as compared with corresponding values calculated for an ideal flare. The increase in V^* has the effect of shifting the point of intersection, that is, the necessary gliding speed, to larger values. Within the range of the calculations made in this paper, which include values of $(L/D)_o'$ as low as 1.25 and values of $\bar{E}_h$ and t_r as high as 1/2 and 1 second, respectively, the point of intersection fell well within the range of normal glide speeds.

The conclusion reached was that, even with $(L/D)_o'$ values as low as 1.25, high values of the stalling speed and steady-glide sinking speed of an airplane would not, in themselves, necessitate the specification of a landing maneuver beyond the capabilities of a pilot.

Ground Run

The next phase of the landing maneuver to be considered is the ground run. If the transition from the flare-path attitude to the ground-run attitude is assumed to take place instantaneously at the point of contact with the ground, reference 5 supplies the following equation for computing the no-wind braking-run distance:

$$s_b = \frac{{V_b}^2}{2g\left(\frac{D_b}{W} - \frac{\mu L_b}{W}\right)} \log_e \frac{\frac{D_b}{W} + \mu - \frac{\mu L_b}{W}}{\mu} \tag{20}$$

Writing equation (20) in another form produces

$$s_b = \frac{W}{\rho g S\left(C_{D_b} - \mu C_{L_b}\right)} \log_e\left[\frac{1}{C_{L_{max}}}\left(\frac{C_{D_b} - \mu C_{L_b}}{\mu}\right) + 1\right] \tag{21}$$

Equation (21) shows that the braking distance s_b varies directly as the wing loading W/S.

Minimum-Length Landing Field

The previously given flare-path and ground-run calculations may be used as a basis for determining the minimum length of the landing field

required for landing any particular airplane safely. One essentially new feature must be added to these calculations. This feature is an allowance for variation in pilot judgment in the horizontal location, with respect to the landing field, of the straight glide which precedes the landing flare.

Let the basic length of field s_{basic} for a particular airplane landing with a given wing loading be the horizontal distance from the point on the still-air minimum-distance safe-landing flare path where the airplane is 50 feet off the ground to the point at the end of the ground run which follows the completion of the landing flare. (See fig. 10.) This landing path, which the pilot probably will try to fly, contains inherent margins of safety to allow for variation in pilot judgment. An allowance must be made for deviations on both sides of this path which are due to variations in judgment. The ideal flare path and the maximum flare path will be the lower- and upper-limit flare paths, respectively, as they were in determining the minimum-distance safe-landing flare region. These limit flare paths now will be horizontally spaced by forward- and rearward-limit glide paths. The result will be a horn-shaped region, flared out on the end by the limiting flare paths, within which a pilot should be able to keep the airplane. The lower limit of the horn would correspond to the ideal flare path which the pilot, after undershooting the intended glide path by an amount equal to the maximum allowable variation in judgment, would find necessary to fly in order to land the airplane successfully. The upper limit of the horn would correspond to the maximum flare path which the pilot would use after overshooting the intended glide path by an amount equal to the maximum allowable variation in judgment.

The accuracy with which a pilot can bring an airplane into a predetermined position from which he begins the landing maneuver proper may be largely affected by the type of approach pattern used. To try to take into account the characteristics of the various approach patterns now in common use is not within the scope of this paper. This analysis is limited to that part of the landing maneuver which is common to most types of landing patterns, namely, a steady glide in line with the runway for a certain period of time, followed by a landing flare and the ground run.

Previously the importance of pilot judgment at the start of a landing flare has been considered. Another point where this judgment is of equal or greater importance is at some particular instant before the pilot starts the flare at which time the pilot must determine whether his glide path is properly oriented with respect to the field. The length of time of this straight glide after the pilot has made his final judgment and no longer will change his glide before starting the flare will depend to a large extent on the type of airplane involved. For a small maneuverable airplane this time would probably be small, say 5 seconds, but for larger airplanes this time of glide would be larger.

It is realized that the judgments involved in orienting the glide path with respect to the runway are primarily angular in nature. In the present analysis, however, it is assumed for ease of calculation that these judgments are based on estimates of height and distance from the field. Since the errors in the estimated distances are assumed to be proportional to the distances estimated, the margins for error allowed are equivalent to angular tolerances.

Call the maximum allowable horizontal variation which a pilot is likely to make in establishing the glide path Δs. This variation is a combination of three errors: an error in distance judgment, an error in altitude judgment, and an error in compensating for wind velocity. The error in distance judgment occurs when the pilot, while trying to establish a steady glide path, estimates the distance from the airplane to some line on the runway or familiar landmark on the ground at a point about where he expects to land. Let the period of steady glide in line with the runway from the time the pilot orients his glide path to the time he begins his flare be known as t_g. Once having established a glide path, the pilot is assumed able to hold a glide speed V_g substantially constant. Then, since the glide-path angle is small so that $\cos \gamma$ may be taken equal to 1, the distance over which the pilot makes his distance judgment is $t_g V_g = s_g$. The error in estimating this horizontal distance then is $E_s s_g$ where E_s is the relative error in estimating the distance s_g and can reasonably be expected to be of the same order of magnitude as E_h. The second error, an error Δh_g in judgment of height, when orienting the glide path may be resolved into a horizontal error. This horizontal variation due to misjudgment of height is $\dfrac{\Delta h_{g_s}}{\tan \gamma_g} = \dfrac{E_h h_{g_s}}{\tan \gamma_g}$, where h_{g_s} is the actual height at this point and E_h is the relative error in height estimation. If h_{g_s} is sufficiently large, it may be preferable for the pilot to rely on the altimeter rather than visual observation to determine his altitude. Under such circumstances, the error in estimating the height at the beginning of the glide would be independent of height and would be associated with altimeter errors. The third error, due to an improper compensation for wind velocity, would be, in effect, the amount of misjudgment of the wind multiplied by the time of landing, that is, $E_v v t_l$ where E_v equals the relative error in judging wind velocity, v equals the wind velocity, and t_l equals the total time of the landing. The horizontal error in the spacing of the glide path Δs is the sum of these three errors:

$$\Delta s = \frac{\Delta h_{g_s}}{\tan \gamma_g} + E_s s_g + E_v v t_l \tag{22}$$

Since s_g and t_l are larger for a high-speed airplane with a high gliding speed, the error Δs would be larger than for a slower airplane making the same type of landing pattern. The error Δs taken in each direction from the glide path which the pilot attempts to fly will determine the horizontal spacing of the limit glide paths. The minimum-length safe-landing-field distance s_T is the horizontal distance from the point at which the undershooting limit flight path is at 50 feet altitude to the end of the ground run on the overshooting limit flight path.

Figure 10 is a schematic diagram of the minimum-length safe-landing field showing the limiting flight paths from the beginning of the straight glide to the end of the ground run. The rectangle shown at the start of the straight glide in this figure represents an imaginary area within which the pilot should be able to establish his straight glide. The type of approach pattern used is assumed to be sufficiently accurate so that it is aerodynamically possible for the pilot to pass through the box at the necessary glide speed. The rectangle is centered around the start of the straight glide preceding the minimum-distance safe-landing flare. The length of the horizontal sides equals $2\left(E_s s_g + E_v v t_l\right)$ and the length of the vertical sides equals $2h_{g_s}$. The diagonally opposite corners of the rectangle space the forward and rearward limit glide paths.

Figure 11 shows the minimum-length safe-landing field for a high-speed airplane with $\frac{W}{S} = 51.9$ pounds per square foot, $C_{L_{max}} = 1.12$, $\rho = 0.002219$ slug per cubic foot, $(L/D)_o = 4.08$, and $V_o = 204.5$ feet per second. Two configurations are shown, one for a t_g of 5 seconds and one for a t_g of 20 seconds. The values of the constants used were $E_s = \frac{1}{4}$, $t_r = 0.5$ second, $\Delta h_{g_s} = 125$ feet, $E_v v = 6$ feet per second, and $t_l = 60$ seconds and 45 seconds. The length of field determined by this method for the case $t_g = 20$ seconds is approximately the same length as the landing field required for this airplane given in figure 4-29 of reference 6. Examination of this figure and figure 11 shows that the factor which has the greatest effect on landing-field length is not the difference between the limit flare paths but the difficulty which the pilot has in starting the glide at the proper spot as shown by the horizontal spread between the glide paths. In order to get into a properly oriented glide path, the pilot must judge effectively large distances, which may lead to appreciable errors.

The calculations presented in the analysis show that an effective way to decrease the size of runway required would be to fix the point of the start of the straight glide. A point of attack for this problem might be the use of high-frequency radio beams to aid in determining the position of the airplane.

No analysis of variable power-on landing flares is presented in this paper since, with the use of power, there are an infinite number of possible flare-path shapes. In general, the use of power results in a variable lift-drag ratio. In the problem of the minimum-length landing field, the use of power may help to shorten considerably the necessary length of field required by decreasing the large horizontal error Δs at the start of the straight glide. By using power, a pilot can approach with a flatter glide path at a lower altitude with an airspeed somewhat less than the steady glide speed for a power-off landing. Then when the pilot is close to the field he can judge his altitude and distance quite accurately. When the power is cut, the landing flare is completed along the power-off flare path indicated by the preceding analysis.

CONCLUDING REMARKS

An analysis of the power-off landing maneuver is presented in which an attempt is made to consider the human capabilities of the pilot in addition to the aerodynamic characteristics of the airplane. It is assumed that the pilot's judgment of distance may be inaccurate by a certain fraction E_h and that a time delay t_r occurs between a decision to correct the airplane attitude and the time that such correction is effected. These parameters E_h and t_r are included in the landing calculations to modify the optimum landing paths derived from purely aerodynamic considerations so as to give them inherent safety margins. The corresponding determination of a minimum safe initial glide speed and the definition of a region within which the pilot should fly in order to make a safe landing in a minimum distance are described.

A qualitative conclusion concerning E_h and t_r was brought out in the analysis. As the stalling speed of the airplane is increased, the effect of the height-estimation factor becomes relatively more important than the time-delay factor.

A conclusion reached as the result of several sample calculations was that, even with values of the lift-drag ratio at the base of the

ideal flare as low as 1.25, high values of the stalling speed and steady-glide sinking speed of an airplane would not, in themselves, necessitate the specification of a landing maneuver beyond the capabilities of the pilot.

The results obtained from an analysis of the present type show the desirability of future research to determine accurate values of such factors as E_h and t_r, and the variables upon which they depend.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., February 11, 1952.

APPENDIX

CALCULATIONS OF AIRSPEED VARIATIONS ALONG AN ARBITRARY FLARE PATH

The variation of airspeed along an arbitrary flare path may be found by the application of the equations of motion of the airplane.

If γ is assumed small so that $\cos \gamma = 1$ and $\sin \gamma = \gamma$, the equation of motion takes the form

$$C_L \frac{\rho}{2} SV^2 - W = \frac{WV^2}{gR} \tag{23}$$

and

$$\frac{W}{g} a_t = -C_D \frac{\rho}{2} SV^2 - W\gamma \tag{24}$$

Solving equations (23) and (24) for V^2 and equating the resulting values produces

$$\frac{1}{C_L \frac{\rho}{2} S - \frac{W}{gR}} = -\frac{\frac{a_t}{g} + \gamma}{C_D \frac{\rho}{2} S}$$

from which

$$a_t = -\frac{gC_D \frac{\rho}{2} S}{C_L \frac{\rho}{2} S - \frac{W}{gR}} - g\gamma \tag{25}$$

But since $a_t = V \frac{dV}{ds}$

$$\frac{dV}{ds} = -\frac{1}{V}\left(\frac{gC_D \frac{\rho}{2} S}{C_L \frac{\rho}{2} S - \frac{W}{gR}} + g\gamma\right) \tag{26}$$

Solving equation (23) for V produces

$$V = \sqrt{\frac{W}{C_L \frac{\rho}{2} S - \frac{W}{gR}}}$$

Substituting this value of V into equation (26) and simplifying gives

$$\frac{dV}{ds} = -\frac{g}{\sqrt{W}}\left(\frac{C_D \frac{\rho}{2} S}{\sqrt{C_L \frac{\rho}{2} S - \frac{W}{gR}}} + \gamma\sqrt{C_L \frac{\rho}{2} S - \frac{W}{gR}}\right) \tag{27}$$

Since s has been taken as the horizontal distance along the corrected flare path rather than the distance along the curve, the radius of curvature of the corrected flare may be exactly expressed as

$$R = \frac{\left[1 + (dh/ds)^2\right]^{3/2}}{d^2h/ds^2}$$

Substituting this value of R into equation (27) gives the following expression for the velocity gradient along the flare path:

$$\frac{dV}{ds} = -\frac{g}{\sqrt{W}}\left\{\frac{C_D \frac{\rho}{2} S}{\sqrt{C_L \frac{\rho}{2} S - \frac{W(d^2h/ds^2)}{g\left[1 + (dh/ds)^2\right]^{3/2}}}} + \gamma\sqrt{C_L \frac{\rho}{2} S - \frac{W(d^2h/ds^2)}{g\left[1 + (dh/ds)^2\right]^{3/2}}}\right\} \tag{28}$$

From the known glide-path speed and corrected flare-path dimensions, the actual lift and drag coefficients and the true velocities along the corrected flare path may be obtained by use of the airplane lift-drag polar curve and the following equations:

$$V = \int \frac{dV}{ds}\, ds$$

$$C_L = \frac{W/S}{\rho/2}\left(\frac{1}{V^2} \div \frac{1}{gR}\right) \qquad \text{(from equation (23))}$$

where

$$R = \frac{1}{dV/ds}$$

In order to reduce the work of calculating the velocity without noticeably imparing the accuracy of the calculations, the following changes may be made in equation (28):

$$\frac{dh}{ds} = \gamma$$

$$\frac{d^2h}{ds^2} = \frac{d\gamma}{ds}$$

Expand $\left(1 + \gamma^2\right)^{3/2}$ in a series and neglect all values of γ of order higher than one. The result is $\left(1 + \gamma^2\right)^{3/2} = 1$. Substituting these relations into equation (28) gives

$$\frac{dV}{ds} = -\frac{g}{\sqrt{W}}\left[\frac{C_D \frac{\rho}{2} S}{\sqrt{C_L \frac{\rho}{2} S - \frac{W(d\gamma/ds)}{g}}} + \gamma\sqrt{C_L \frac{\rho}{2} S - \frac{W(d\gamma/ds)}{g}}\right] \qquad (29)$$

If the preceding relations are used, a speed check may be made along an arbitrary flare path by a step-by-step process for increments of ds. Begin with the assumed speed of entrance into the flare path. A curve of γ against s can be plotted from figure 2 and a curve of $d\gamma/ds$* can be obtained from equation (3). The drag coefficient C_D may be obtained from the lift-drag polar curve for the airplane. The amount of speed correction needed is found by beginning at the base of the flare path with the stalling speed and using a step-by-step process to solve equation (29) for increments of ds. This procedure gives a curve of V/V_o' against γ for the maximum flare path. The value of V/V_o' at the point of intersection of this curve and the curve of V/V_o' against γ_g derived from the airplane polar determines the value of the glide speed at which the pilot wants to enter the flare.

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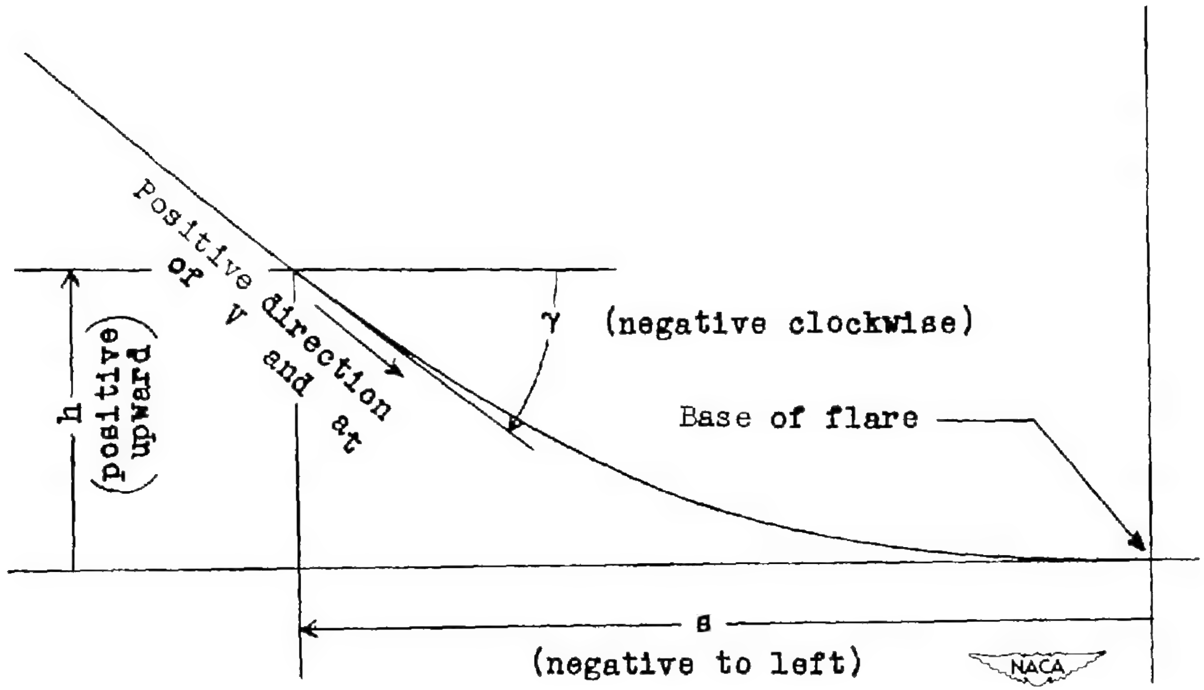


Figure 1.- Sign conventions used in landing-flare derivation.

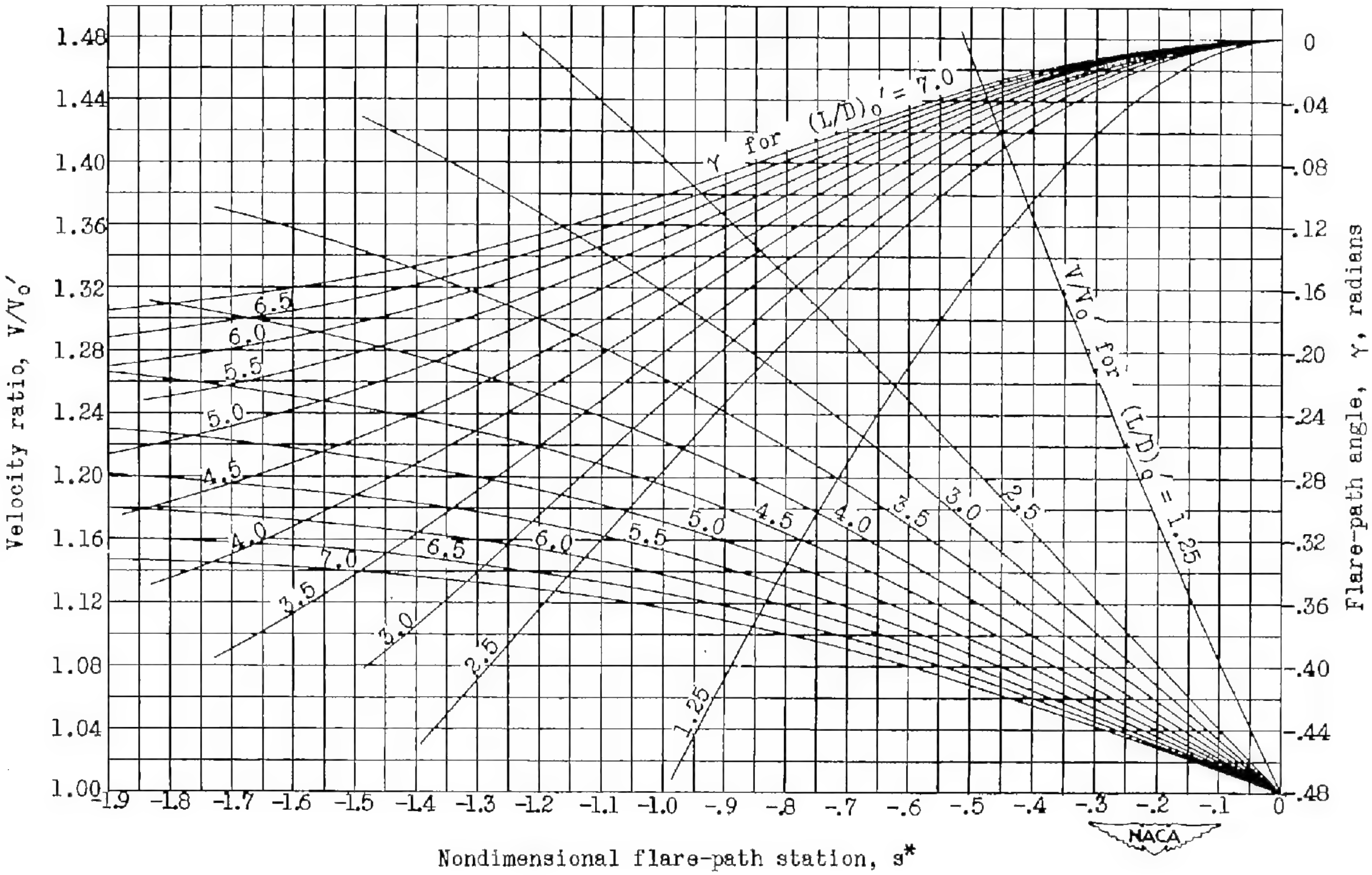


Figure 2.- Ideal-flare-path basic-parameter curves for several values of $(L/D)_o'$.

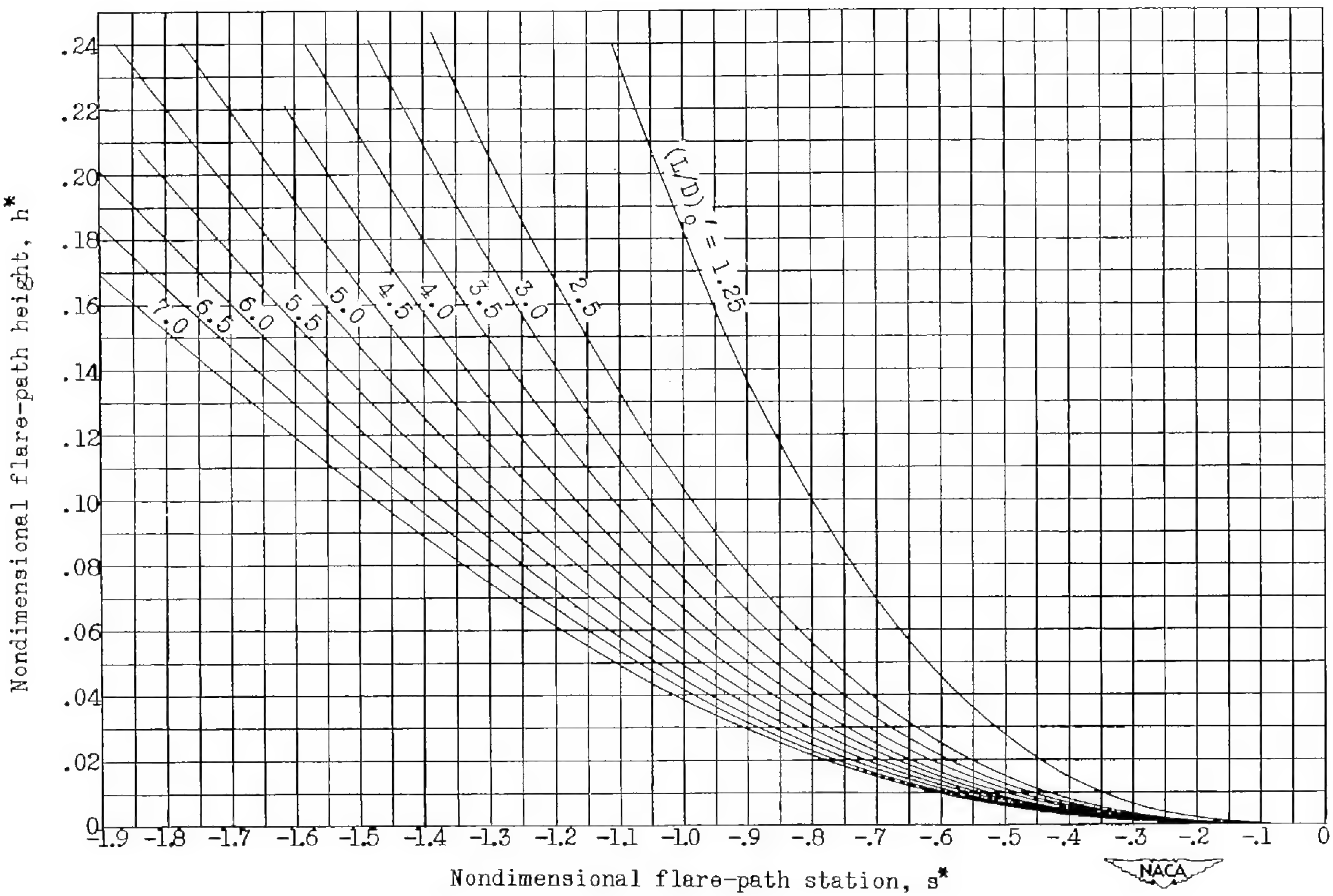


Figure 3.- Nondimensional ideal flare paths for several values of $(L/D)_o'$.

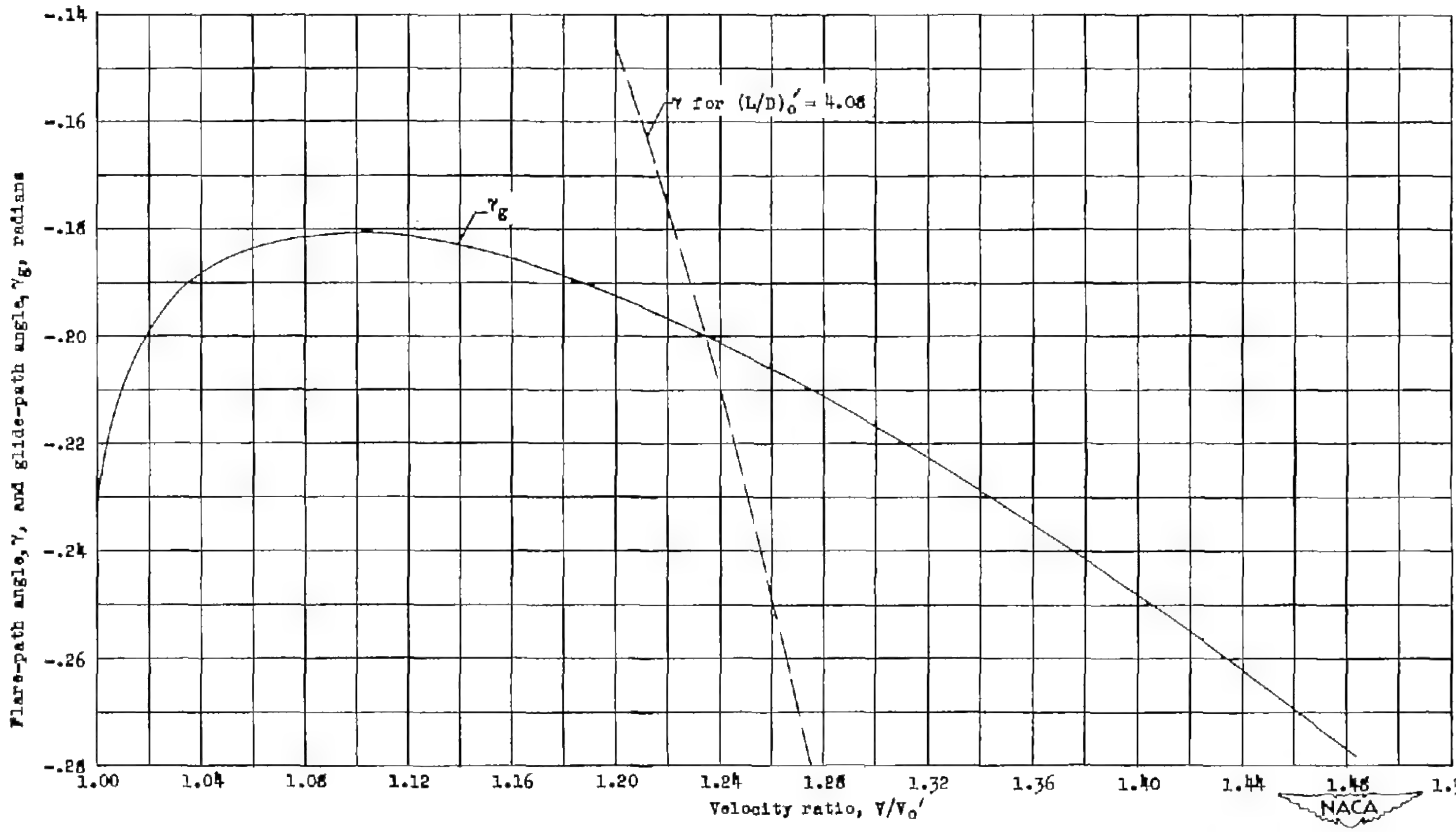
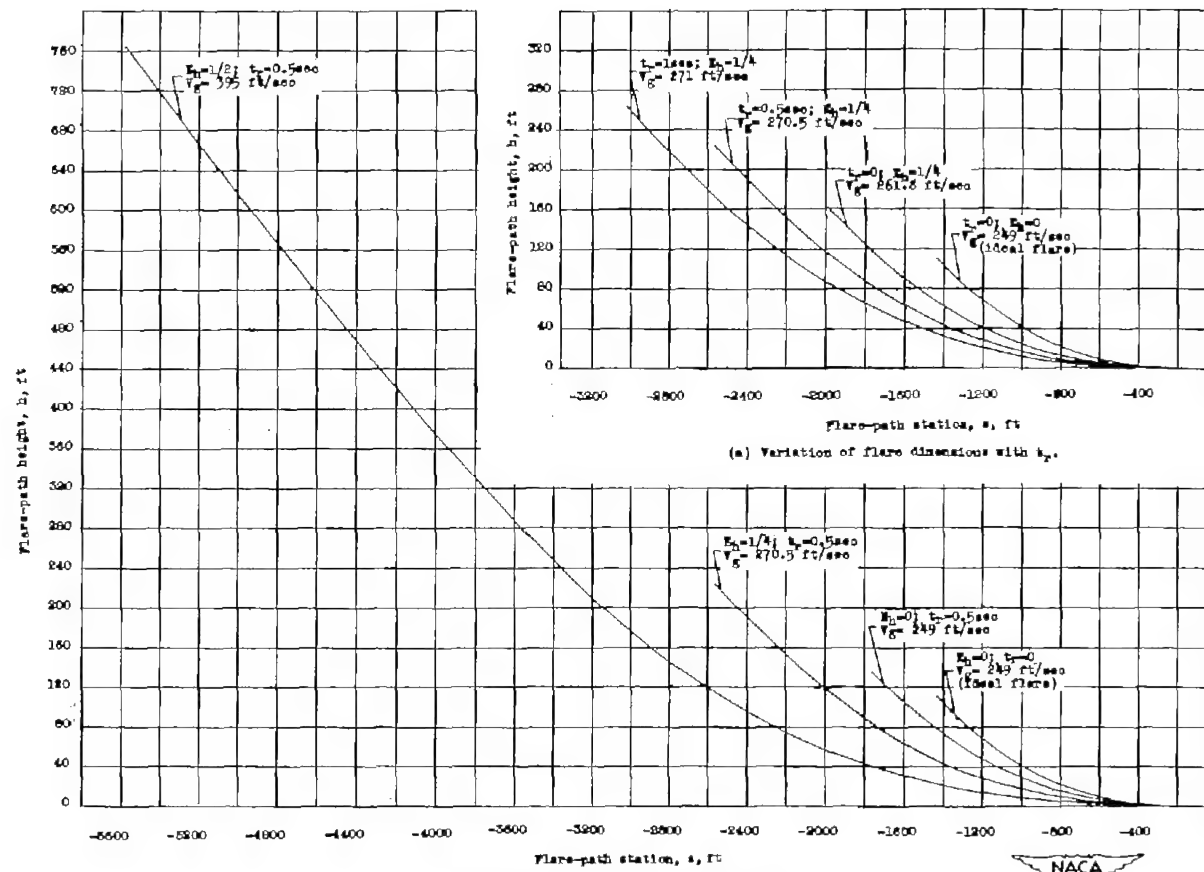


Figure 4.- Determination of glide-path conditions for a particular high-speed airplane with an $(L/D)_o'$ value of 4.08.



(a) Variation of flare dimensions with t_r.

(b) Variation of flare dimensions with E_h.

Figure 5.- Variation of flare dimensions with height-estimation error and time delay for a high-speed airplane. $\frac{W}{S}$ = 51.9 pounds per square foot; $C_{L_{max}}$ = 1.12; $(L/D)_o$ = 4.08; V_o = 204.5 feet per second; ρ = 0.002219 slug per cubic foot.

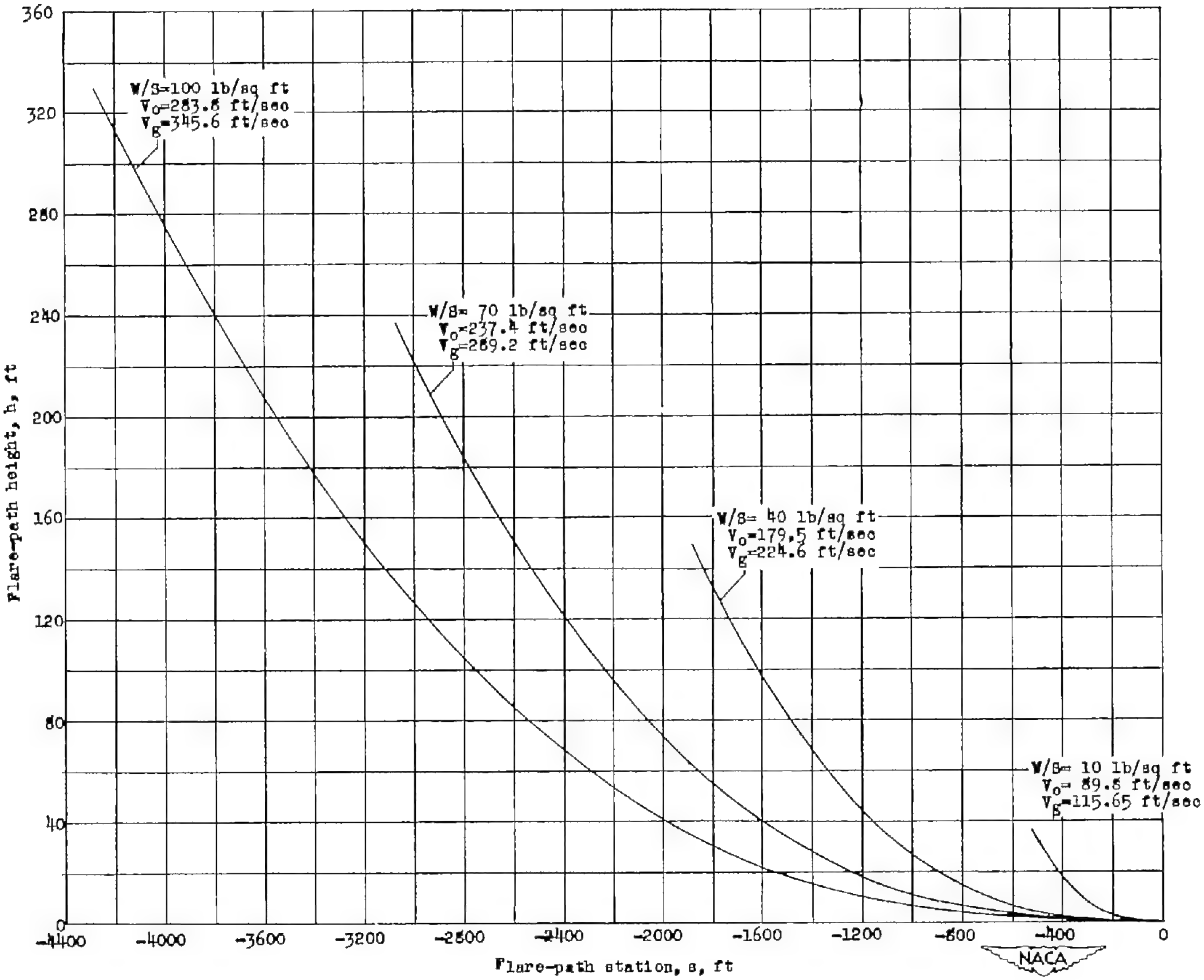


Figure 6.- Effect on dimensional flare path of varying the wing loading of a high-speed airplane. $C_{L_{max}} = 1.12$; $(L/D)_o = 4.08$; $\rho = 0.002219$ slug per cubic foot; $E_h = \frac{1}{4}$; $t_r = 0.5$ second.

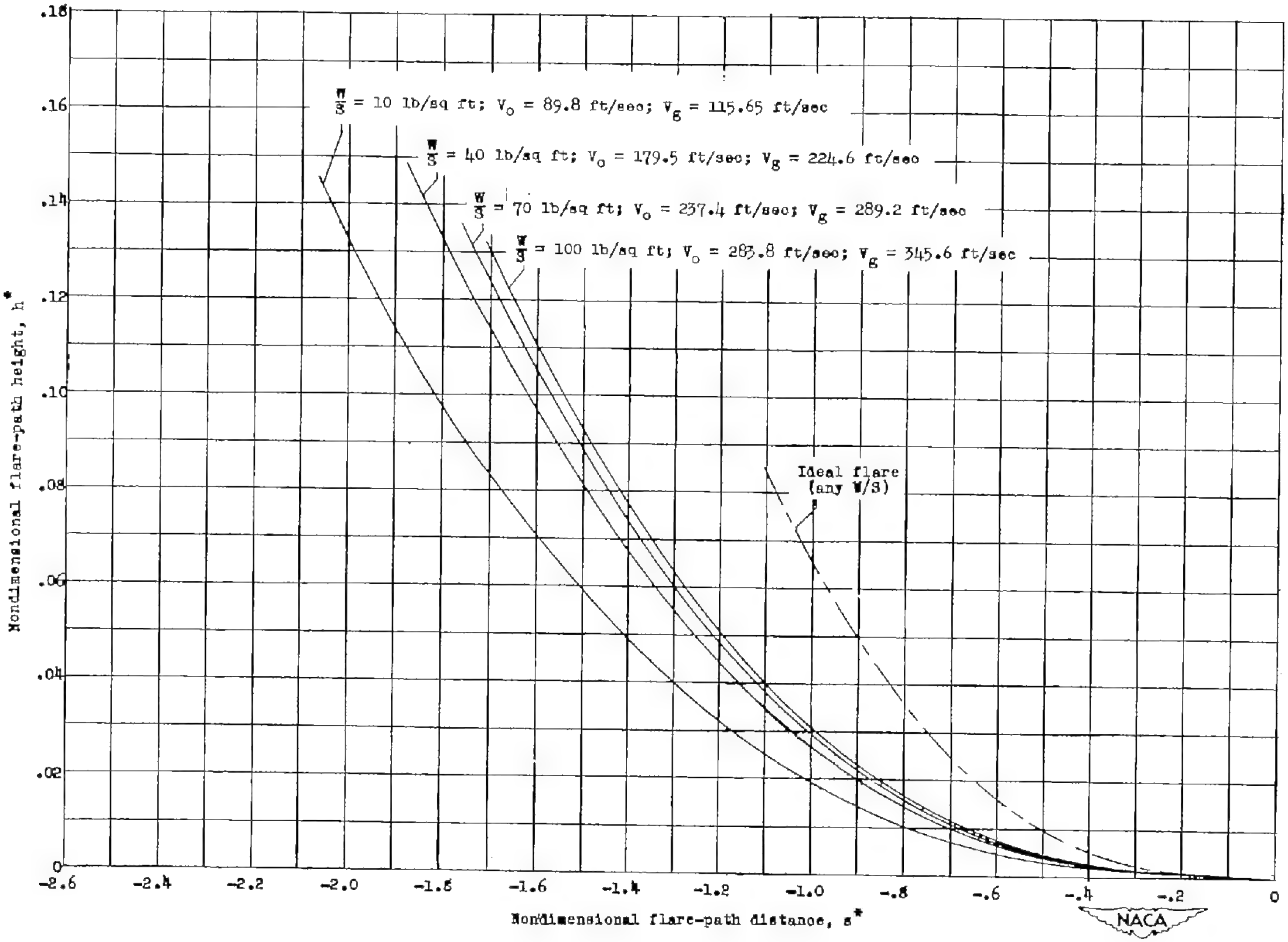


Figure 7.- Effect on nondimensional flare path of varying the wing loading of a high-speed airplane. $C_{L_{max}} = 1.12$; $(L/D)_o = 4.08$; $\rho = 0.002219$ slug per cubic foot; $E_h = \frac{1}{4}$; $t_r = 0.5$ second.

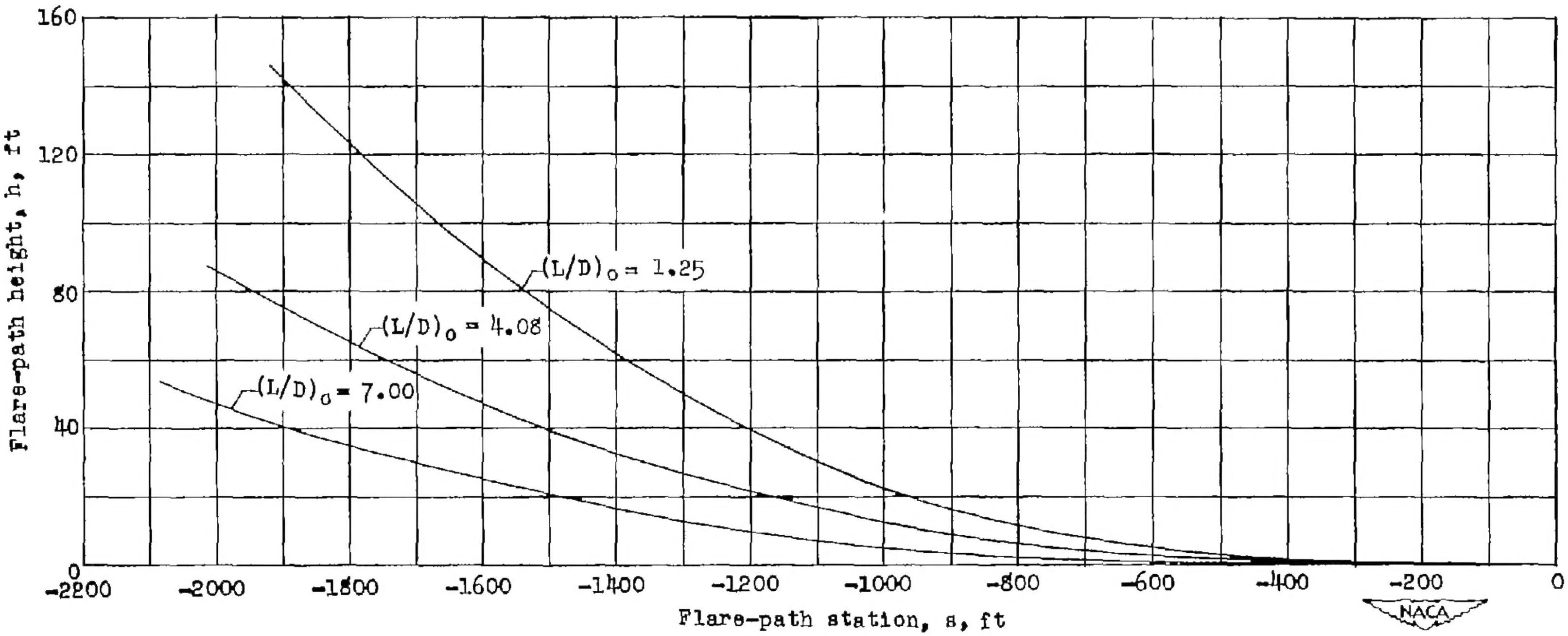


Figure 8.- Effect of variation of lift-drag ratio on flare-path dimensions as shown by three airplanes with different values of $(L/D)_o$ but with wing loadings adjusted to give each airplane a stalling speed of 225 feet per second. $E_h = \frac{1}{4}$; $t_r = 0.5$ second.

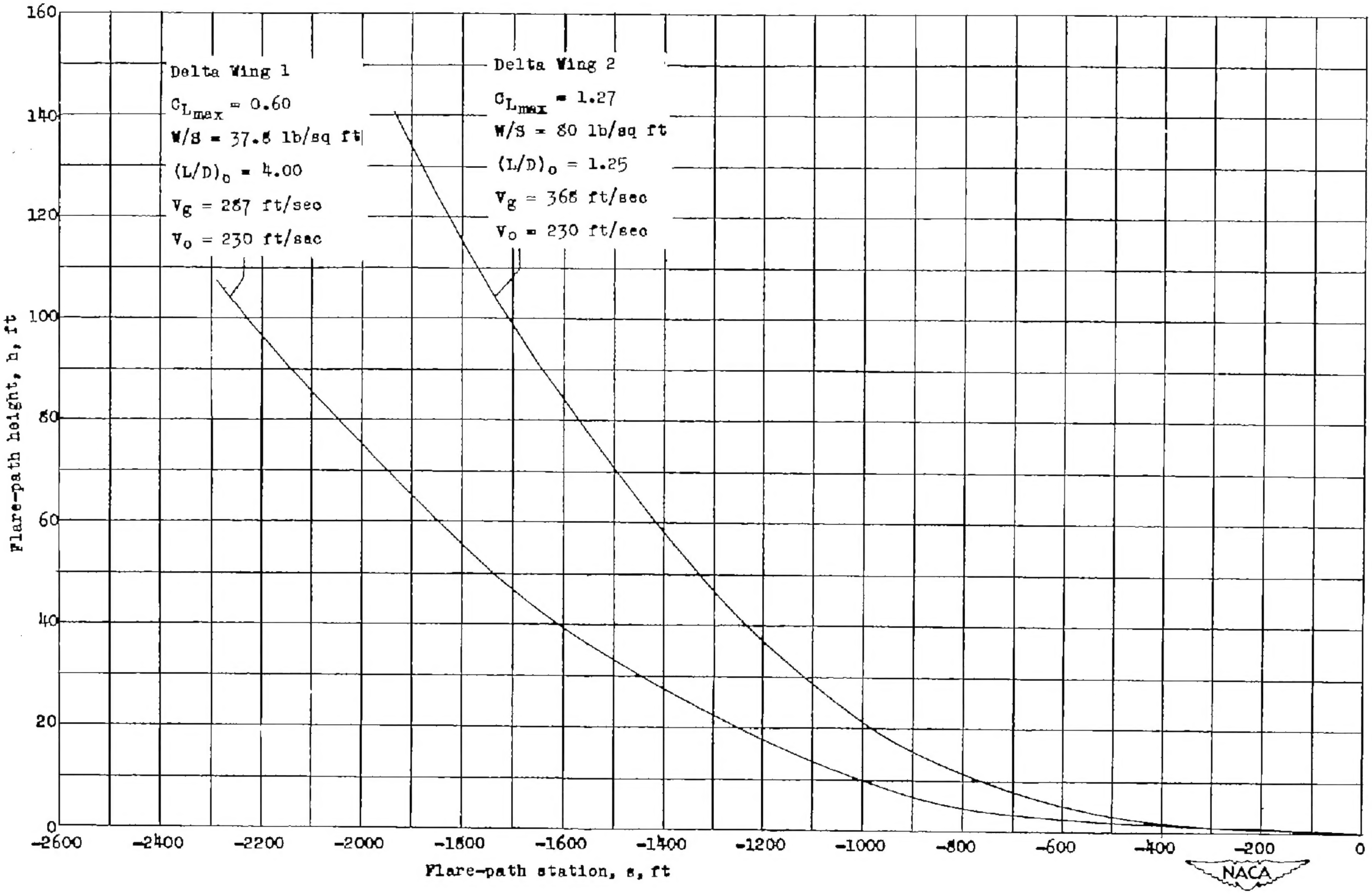


Figure 9.- Landing flare paths for two delta-wing configurations loaded to a stalling speed of 230 feet per second. $E_h = \frac{1}{4}$; $t_r = 0.5$ second.

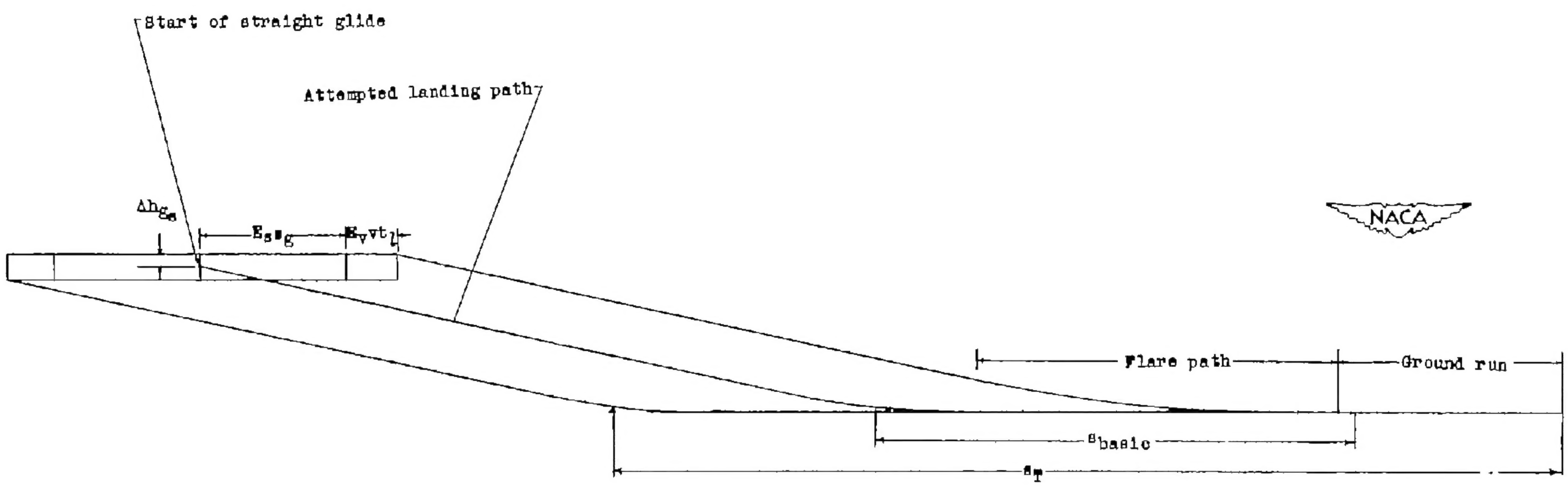


Figure 10.- Schematic breakdown of minimum-length landing field and limiting flight paths.

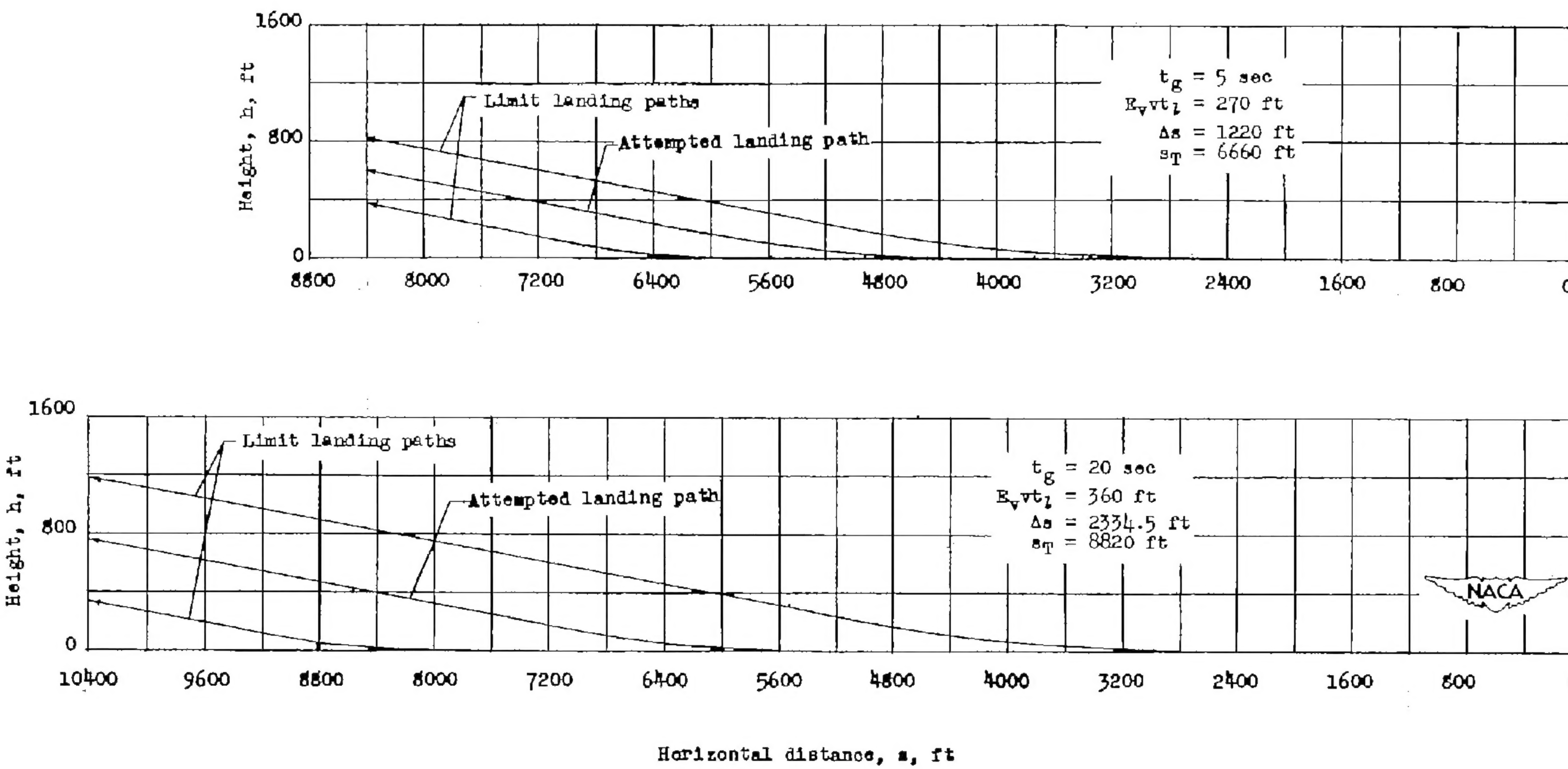


Figure 11.- Minimum-length landing field and part of limiting flight paths for a high-speed airplane for two values of t_g. $C_{L_{max}} = 1.12$; $(L/D)_o = 4.08$; $E_h = \frac{1}{4}$; $t_r = 0.5$ second; $\rho = 0.002219$ slug per cubic foot; $\frac{W}{S} = 51.9$ pounds per square foot; $V_o = 204.5$ feet per second; $\Delta h_{g_s} = 125$ feet; $s_{basic} = 4595$; $\frac{\Delta h_{g_s}}{\tan \gamma_g} = 622$.